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Mission Capability Gains from Multi-Mode Propulsion Thrust Profile Variations for a Plane Change Maneuver

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Plane change maneuvers are very costly in terms of propellant required. Performing a plane change can quickly drain a spacecraft's propellant required. The utilization of a multi-mode propulsion (MMP) system could greatly reduce the amount of propellant required for the maneuver. MMP refers to a propulsion system that couples an electric and chemical propulsion system which utilize the same propellant and piping¹. A fully coupled system provides many advantages over a spacecraft with a single chemical or electric propulsion system^{2,3}. The trade space created by the utilization of a MMP system as compared with only a chemical or electric propulsion system is described as well as the effect of various thrust profiles on a plane change mission. The spacecraft analyzed had a mass of 180 kg and was required to perform a 15 deg plane change within 90 days⁴. The chemical system alone required more than the 80 kg of available propellant to complete the maneuver. Numerous thrust profiles were analyzed for the electric propulsion system, as well as a combined electric/chemical system. Development of equations and the analysis method are described. The electric propulsion system thrusting constantly was able to achieve the plane change within the time limit.

Nomenclature

i	= inclination [rad]
I_{sp}	= specific Impulse [s]
m_{init}	= initial mass of the spacecraft [kg]
m_{final}	= final mass of the spacecraft [kg]
m_p	= propellant mass [kg]
m_{sc}	= mass of the spacecraft [kg]
MMP	= multi-mode propulsion
\vec{r}_{ijk}	= position in the Geocentric Equatorial Reference Frame [km]
\vec{r}_{pqw}	= position in the Perifocal Reference Frame [km]
R_1	= rotation matrix about x-axis
R_3	= rotation matrix about z-axis
t_{burn}	= thrust burn time [s]
T_{sc}	= thrust of the spacecraft [N]
\vec{v}_{diff}	= vector between current and final velocity vector
$\vec{v}_{ijk\ final}$	= velocity vector in the Geocentric Equatorial Reference Frame of spacecraft in intended orbit [km/s]
$\vec{v}_{ijk\ trans}$	= velocity vector in the Geocentric Equatorial Reference Frame of spacecraft in transfer orbit [km/s]
ΔV	= change in velocity [km/s ²]
δ	= desired plane change [rad]
ω	= argument of perigee [rad]
Ω	= right argument of the ascending node [rad]

I. Introduction

The spacecraft initially had a mass of 180 kg with 80 kg available for propellant. The spacecraft was located in a circular orbit at an altitude of 500 km. The destination orbit was circular with an altitude of 500 km and

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inclination of 15 deg. The plane change needed to be completed in less than 90 days. The thrust vector direction was calculated such that the transfer orbit remained essentially circular.

The initial orbit and the final orbit with an inclination of 15 deg are depicted in Figure 1 in the Geocentric Equatorial reference frame.

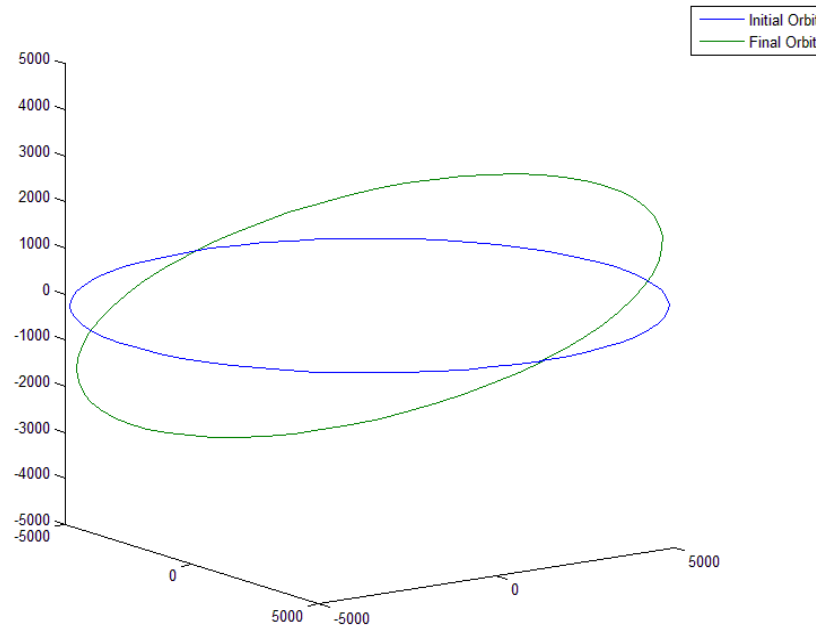


Figure 1. Geocentric Equatorial Reference Frame

Figure 1 shows a scaled version of the initial and final orbit in the Geocentric-Equatorial reference frame and the two crossover points that occur. The electric propulsion system was modeled initially to thrust at the intersection (crossover) locations. The crossover point was where the chemical propulsion system performed the impulsive burn.

A rough schematic of a multi-mode propulsion (MMP) system is shown in Figure 2.

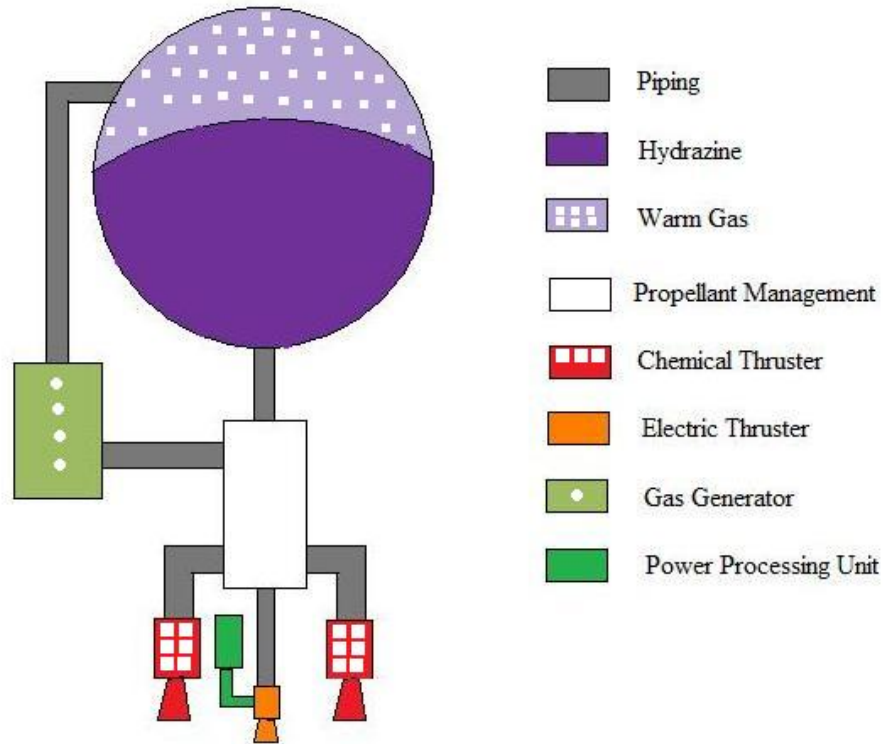


Figure 2. Multi-Mode Propulsion System Schematic

As shown in Figure 2, the MMP system modeled utilized hydrazine. The chemical propulsion system had a specific impulse of 235 sec and a thrust of 22 N. The electric propulsion system had a specific impulse of 600 sec and a thrust of 0.11 N. The mass of the propulsion system was not specifically modeled. The spacecraft was treated as a dry mass of 100 kg that contained all spacecraft components. The propellant limit of 80 kg was treated as the maximum amount of propellant available for the plane change maneuver. The maneuver which would achieve the plane change with the least propellant necessary was considered optimal.

II. Development of Equations

The position and velocity could not be treated as the magnitude of those values for a plane change maneuver. The velocity vector of the spacecraft was broken into a transverse and a radial component^{4,5,6,7}. The chemical propulsion system could perform an impulsive maneuver which is also called a cranking maneuver. The change in velocity required is represented in the following equation,

$$\Delta V = 2V \sin\left(\frac{\delta}{2}\right) \quad (1)$$

Eq. (1), is intended for a large impulse, which excludes an electric propulsion transfer. The resulting delta v from this equation was used as if it were applicable for an electric propulsion system to compare the resulting propellant mass with more exact calculations.

The equation used to calculate the propellant required based on delta v is:

$$m_p = m_{init} \left(1 - e^{\left(\frac{-\Delta V}{I_{sp} g} \right)} \right) \quad (2)$$

Eq. (2), resulted in 103.99 kg of propellant required which exceeded the propellant available for the maneuver. Utilizing this same equation for an electric propulsion system resulted in 51.52 kg of propellant required due to the higher specific impulse.

The electric propulsion system utilized a different equation for the calculation of the propellant mass which did not include the change in velocity term. The electric propulsion system calculated the change in velocity as depicted in Eq. (7).

$$m_{final} = m_{init} - t_{burn} \left(\frac{T_{sc}}{I_{sp}g} \right) \quad (3)$$

Eq. (7) was derived from the commonly used equation for mass flow rate. The electric propulsion system was modeled by transferring the spacecraft position and velocity vector between the Perifocal and the Geocentric-Equatorial reference frame. The position and velocity vectors were transferred into the Geocentric-Equatorial reference frame through the use of the following rotation matrices³.

$$\vec{r}_{ijk} = \vec{r}_{pqw} R_3(\Omega) R_1(\pi - i) R_3(\omega) \quad (4)$$

The same process was used to transform the velocity vector. The direction of the necessary velocity vector may be calculated from this new form of the velocity vector.

$$\vec{v}_{ijk\ diff} = \vec{v}_{ijk\ final} - \vec{v}_{ijk\ trans} \quad (5)$$

The velocity vector was made non-dimensional with the Eq. (6),

$$\vec{v}_{diff} = \frac{\vec{v}_{ijk\ diff}}{|\vec{v}_{ijk\ diff}|} \quad (6)$$

The straight brackets in Eq. (6) represent the magnitude of the vector. The change in velocity depends on the specific impulse, gravity, and the change in mass due to thrusting as shown in Eq. (7)⁵.

$$\Delta v = -I_{sp}g \ln \left(\frac{m_{final}}{m_{init}} \right) \quad (7)$$

In the Eq. (7), m_{init} refers to the mass of the spacecraft prior to the burn and m_{final} to the mass of the spacecraft after the burn. The change in velocity vector was calculated as shown in Eq. (8).

$$\overrightarrow{\Delta v} = \Delta v * \vec{v}_{diff} \quad (8)$$

Eq. (8) was used to update the spacecraft's position and velocity vector and substituted into Eq. (9).

$$\begin{aligned} \vec{r}_{ijk} &= \vec{r}_{ijk} + \overrightarrow{\Delta v} * t_{burn} \\ \vec{v}_{ijk} &= \vec{v}_{ijk} + \overrightarrow{\Delta v} \end{aligned} \quad (9)$$

These equations were used to update the position and velocity vector.

III. Results

Several analysis methods were applied to compare the difference in trip time and required propellant mass that resulted from incrementally verses instantaneously updating the change in velocity and position of the spacecraft. The second part of the analysis compared various thrust durations for the electric propulsion system with the chemical propulsion system performing the remainder of the plane change when the time constraint was reached.

The change in velocity required by the chemical system resulted in a delta v of 1987.31 m/s with 103.99 kg of propellant. The electric propulsion system was analyzed with several different thrust schemes and analysis techniques. The first method was to treat the change in velocity over a period of time as though it had an instantaneous effect on the velocity of the spacecraft. That method of analysis was compared with updating the spacecraft's position and velocity vector every minute for a period of 15 min beginning at the crossover point, as well as centered on the crossover point and updated every few seconds. The final electric propulsion thrust method analyzed was for the electric propulsion system to thrust continuously.

Using these analysis techniques the spacecraft was able to reach the necessary 15 deg plane change within the time constraint for some of the thrust profiles. The results for all these methods and additional trade studies are described in Table 1 through Table 3 and the following sections. The transfer orbit for all the electric propulsion thrust profiles had the same general flight profile shown in Figure 3.

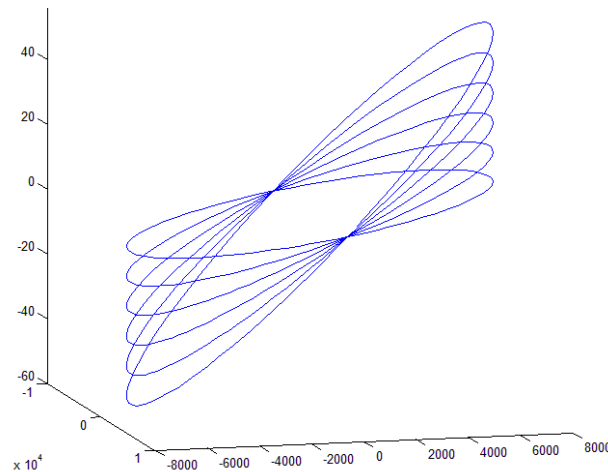


Figure 3. Generic Flight Profile

Figure 3 depicts an electric propulsion transfer orbit. The thrust vector between the transfer orbit and the final orbit were calculated each orbit prior to applying the increase in velocity from the electric propulsion system. Doing this caused the transfer orbit to change inclination slightly, but not change the other orbital parameters. The crossover points were the same for each orbit during the transfer orbit.

A. Single Burn Profile

The position and velocity of the spacecraft during the transfer orbit were updated instantaneously, incrementally, and continuously. The instantaneous change in position was the least realistic of all the orbit transfers modeled. The location of the instantaneous thrust implementation may be seen in Figure 4.

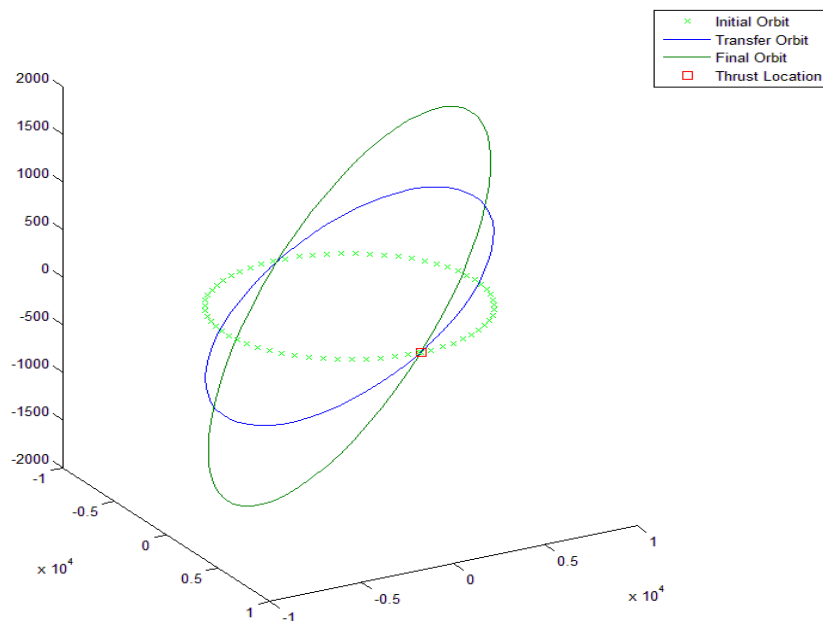


Figure 4. Electric Propulsion System Impulsive Burn

The red square in Figure 4 represents the location where the change in velocity from the 15 min thrust was applied. The effect of the change in velocity on the position and velocity vector were applied instantaneously at the crossover point. When viewed from the top down the orbits appear circular; the elliptical look resulted from the axis of the plot. One 15 min burn achieved about half the desired plane change. This analysis method was compared against two other orbit updating techniques described next.

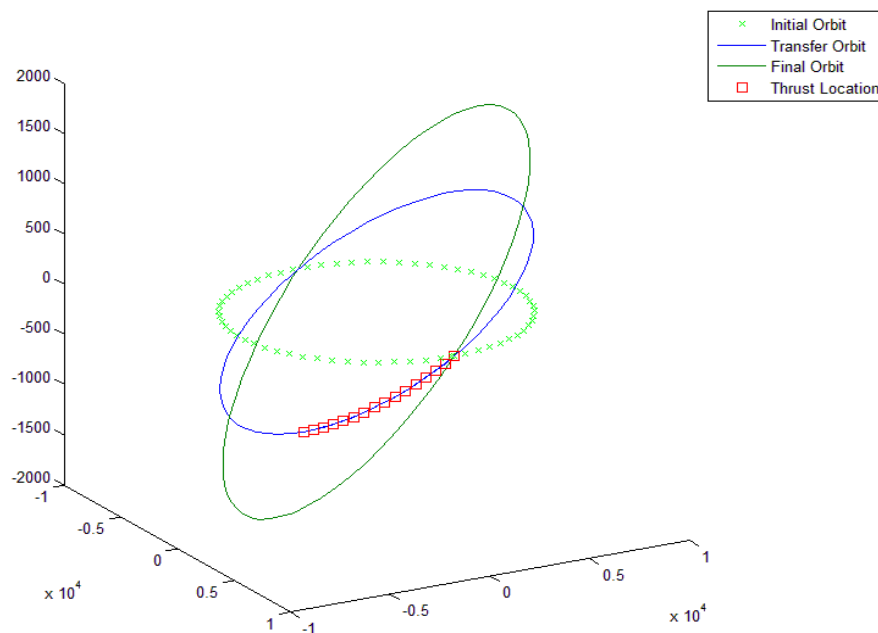


Figure 5. One Burn Beginning at Crossover Point

The thrust profile depicted in Figure 5 involved the transfer spacecraft thrusting each orbit beginning at the crossover point and lasting for 15 min. The spacecrafts position and velocity vector were updated continuously

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during the thrust portion. The results for this thrust profile are shown in Table 1. A slight variation on this thrust profile is shown in Figure 6.

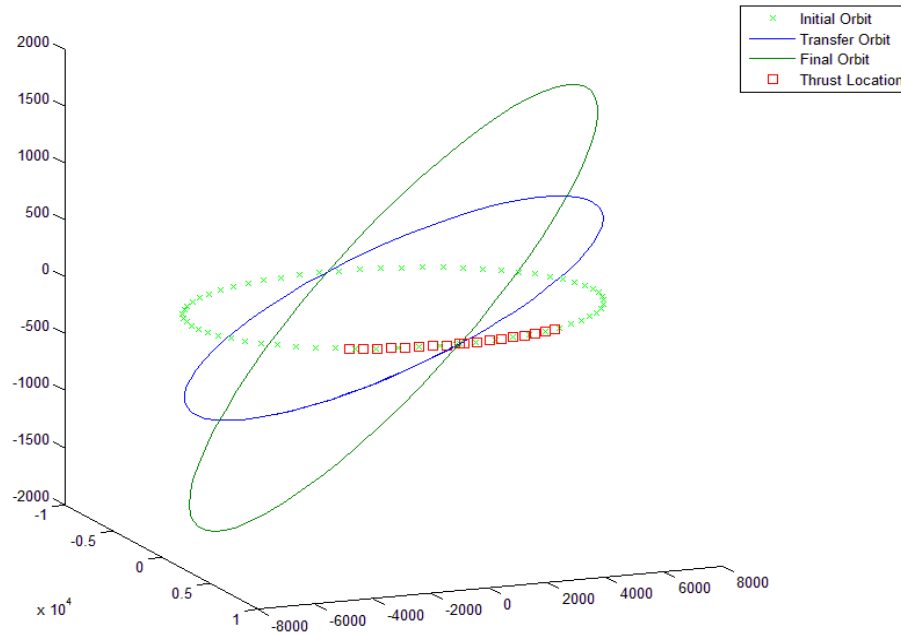


Figure 6. One Burn Centered at Crossover Point

Figure 6 shows the thrust location for a 15 min duration segment over the crossover point. Centering the thrust on the crossover point resulted in a more optimal thrust location with less propellant and a larger inclination angle achieved.

Table 1. One Burn Analysis Method

	Mission Req's	Electric System instantaneous burn	Thrust centered on crossover pt	Thrust beginning at crossover pt
Thrust Duration (min)	N/A	N/A	15	15
Plane Change (deg)	15	7.39	5.70	5.06
Propellant Mass (kg)	< 80	27.65	22.84	23.04
Trip Time (days)	< 90	90.00	90.00	90.00
Delta V (m/s)	N/A	981.63	798.8	806.1

As seen in Table 1 the largest plane change resulted from the instantaneous burn but this method was also the least accurate. Updating the spacecraft position and velocity vector continuously resulted in a slower transfer. Having the thrust location centered on the crossover point resulted in a more optimal transfer than having the thrust begin at the crossover point.

B. Two Burns

The spacecraft was able to achieve the plane change faster through centering the thrust segments on the crossover points. The thrust durations at the crossover points were varied. If the electric propulsion system was unable to achieve the 15 deg plane change the chemical propulsion system performed the remainder of the mission. Two analysis techniques were also modeled, one which updated the position and velocity vector continuously and one that assumed smaller “instantaneous” changes in position and velocity. Figure 7 shows where the thrust was located for the “instantaneous” impulse cases.

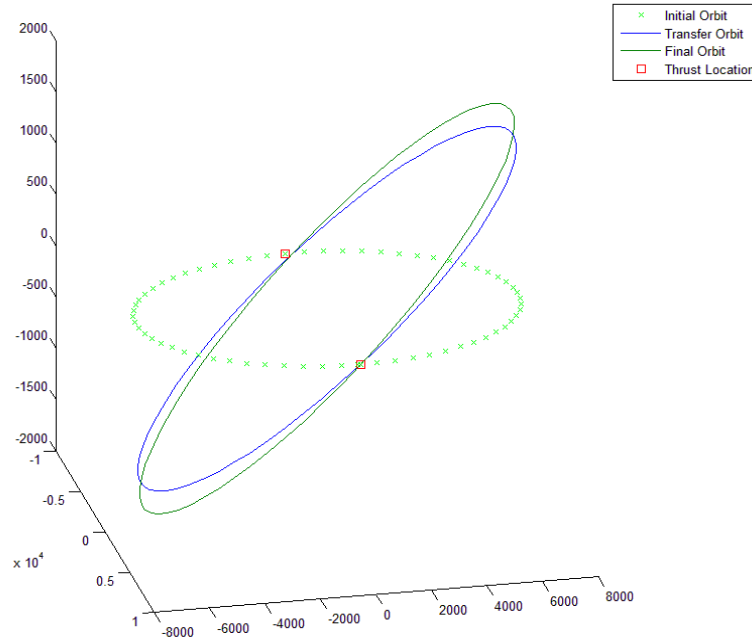


Figure 7. Electric Propulsion System Two Instantaneous Burns

The transfer orbit in Figure 7 with two instantaneous 15 min burns achieved a much larger plane change than the single burn in Figure 4. The total change in the position and velocity vector were applied at the thrust location. The transfer orbit, as seen in Figure 7, was 2 deg short of the 15 deg plane change. The results for this thrust profile are shown in Table 2 .

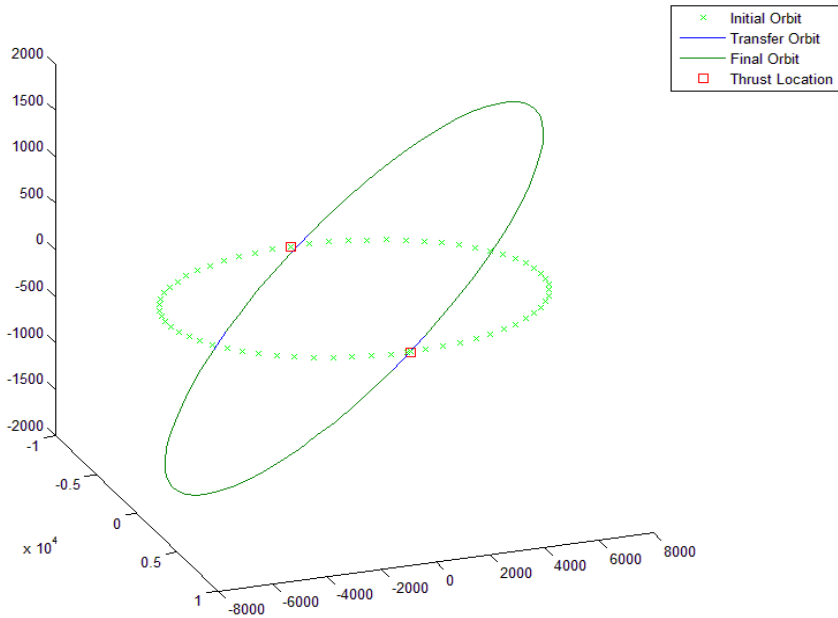


Figure 8. Combined System Orbit Profile

The combined system profile initially utilized electric thrust with two instantaneous 15 min burns then utilized chemical thrust to complete the maneuver. This thrust profile was able to reach the final orbit in the 90 day time limit. This coupling made the final orbit achievable. The results for this thrust profile are shown in Table 2.

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Table 2. Two Burn Analysis Method

	Mission Req's	Electric System 2 instantaneous burns	Thrust beginning at crossover pt	Thrust centered on crossover pt
Thrust Duration (min)	N/A	N/A	15	15
Plane Change (deg)	15	13.01	10.93	12.43
Propellant Mass (kg)	< 80	46.08	46.07	45.97
Mission Time (days)	< 90	90.00	90.00	90.00
Delta V (m/s)	N/A	1740.71	1740.3	1735.9

As seen in Table 2 the instantaneous burn method over-predicted the plane change amount and under-predicted the required propellant. The final thrust profiles that were modeled are the same as those from Figure 5 and Figure 6 except that the burns occurred at each crossover point. The thrust durations were varied from thrusting for 8 min at each crossover point to thrusting continuously.

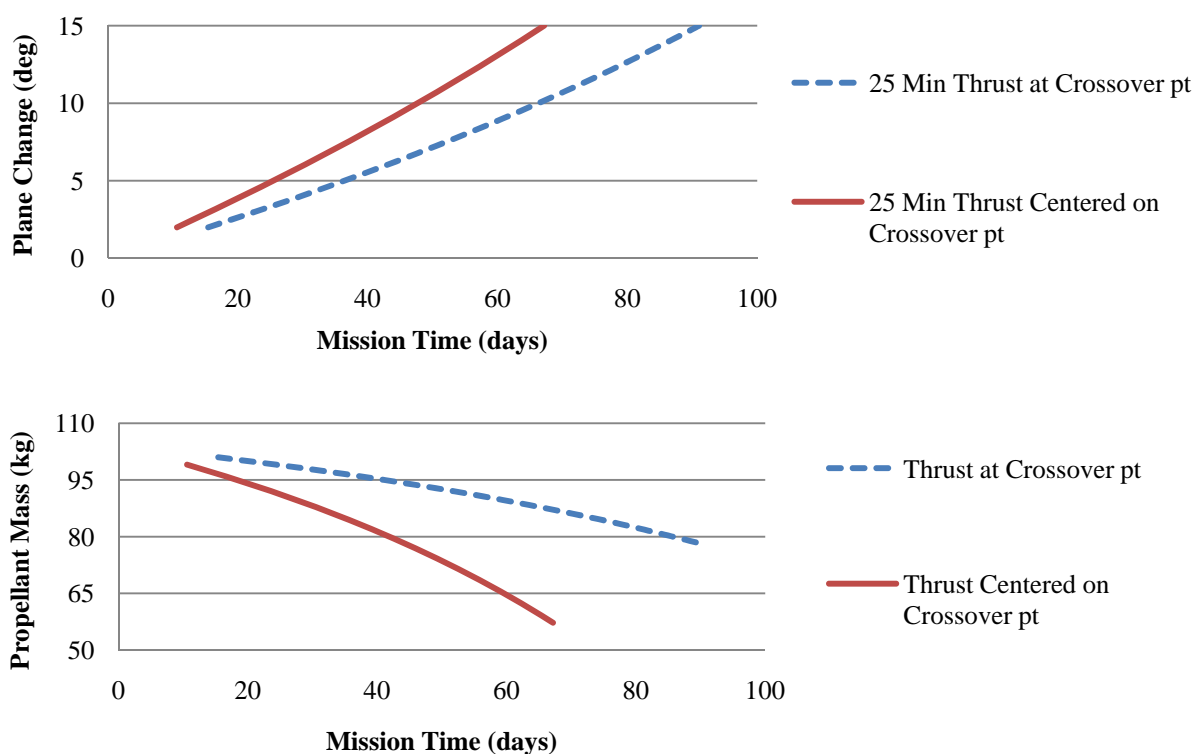


Figure 9. Effect of Thrust Location

The top plot in Figure 9 represented the plane change that was achieved by the use of only the electric propulsion system when the mission time was reached with the two different thrust profiles. The bottom plot represents the combined system with the chemical system performing the remainder of the plane change. The propellant mass in the bottom plot represents a combination of the electric and chemical propellant required to complete the 15 deg plane change. As can be seen from Figure 9, thrust beginning at the crossover point resulted in a longer transfer time and larger propellant required than when the thrust was centered on the crossover point. Thrust duration and required propellant had a direct correlation. Increased thrust duration resulted in increased trip time. This relationship may be seen in Figure 10.

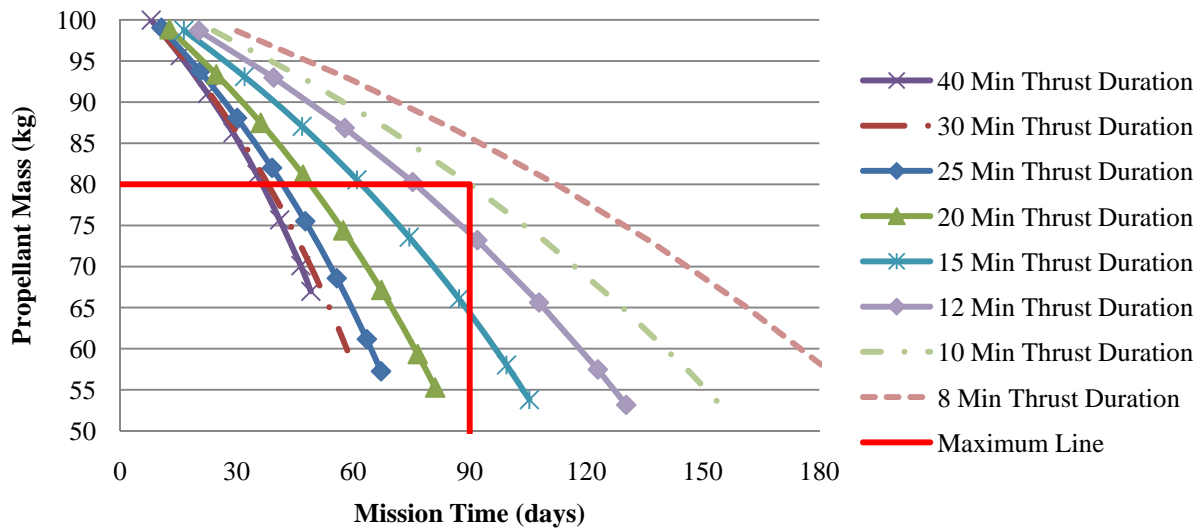


Figure 10. Combined System 15 Deg Plane Change

The combined system shown in Figure 11 represented a system where the electric propulsion system utilized the majority of the mission time to perform the plane change maneuver and the chemical system performed the remainder of the plane change. The red box in the lower left corner of Figure 10 represents the portion of the different thrust duration profiles which would be able to achieve the 15 deg plane change in the time and propellant constraint. As the thrust duration decreased the trip time increased and the propellant mass decreased. The required propellant decreased the most for the longer duration thrust profile. The time required also decreased substantially with the increase in thrust duration per orbit. The propellant mass decreased slightly but after a time of about 110 days the propellant mass savings was minimal. Figure 11 displays the relationship of the thrust duration for a 15 deg plane change in a 90 day time constraint.

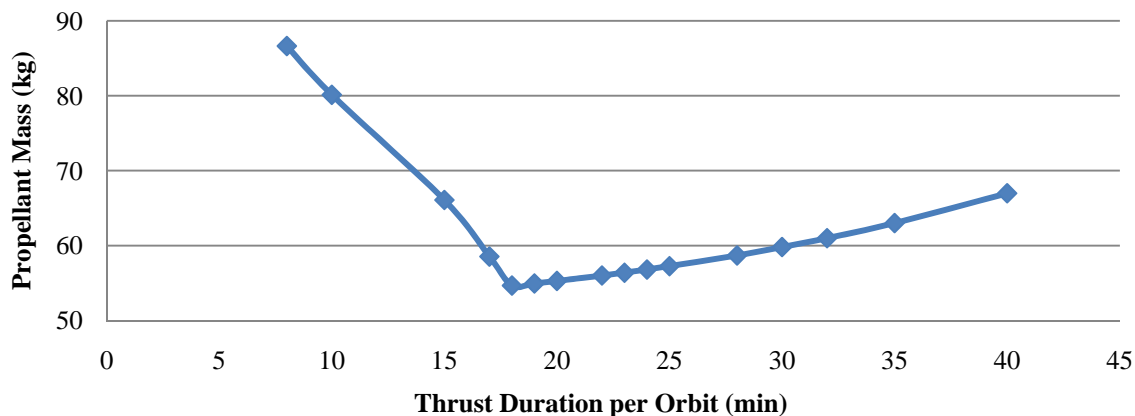


Figure 11. 15 Deg Plane Change Trade Space

The minimum propellant required for a 15 deg plane change in the time constraint was achieved by thrusting at each crossover point for around 18 min. Thrusting for 18 min resulted in 54.66 kg of propellant. The propellant mass required decreased as the thrust duration decreased until 18 min. This was caused by the plane change not being able to be completed entirely by the electric propulsion system. The assist of the chemical propulsion system caused the sharp increase in propellant required. If the maximum trip time were increased the minimum propellant point in Figure 11 would continue to move down and to the left. Figure 12 displays this for a variety of thrust durations.

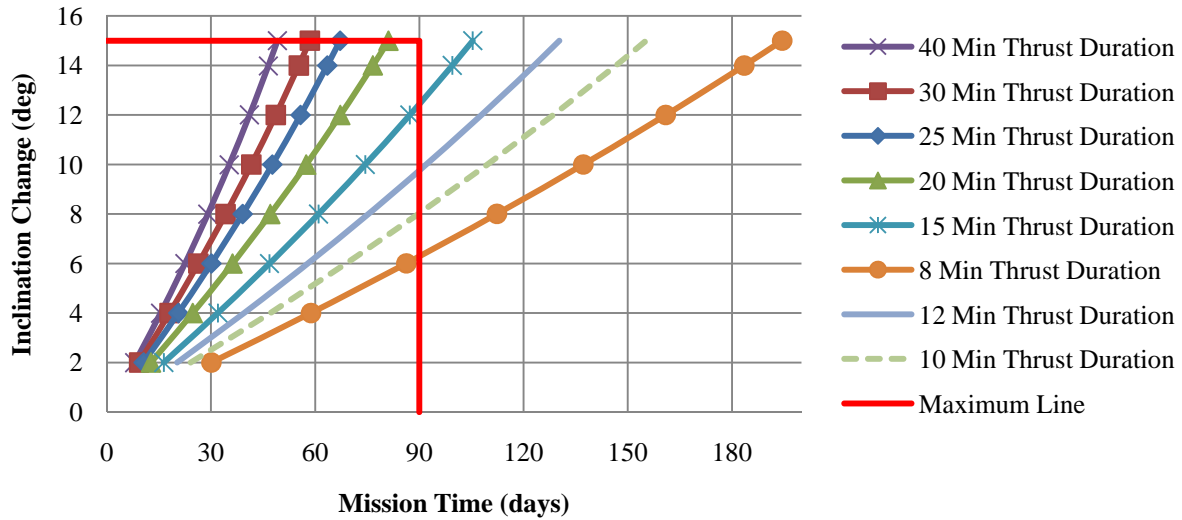


Figure 12. Inclination Change and Mission Time Relationship

As can be seen from Figure 12, the trip time greatly increased as the thrust duration decreased. The variance in time was much smaller for smaller plane change angles. The difference in time ranges from around 22 days for a 2 deg inclination change to around 150 days for a 15 deg plane change.

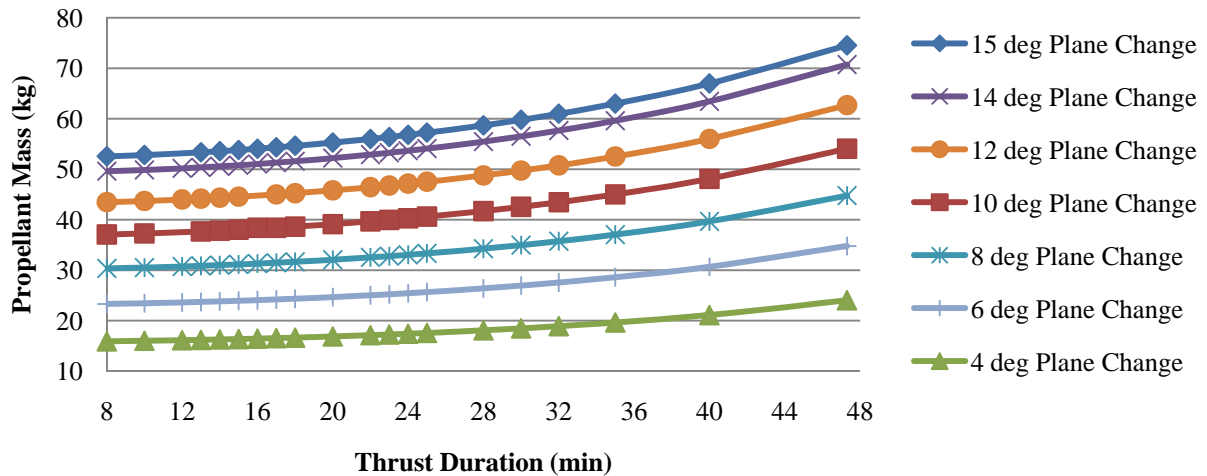


Figure 13. Required Propellant and Thrust Duration Relationship

As seen from Figure 13 the curves representing the propellant mass with respect to thrust duration were very uniform. The spacing between the lines decreased slightly as the plane change increased. For each of the plane change amounts the propellant mass savings through longer thrust durations had a smaller payoff. The slope of all the trend lines flattened out until the mass savings was less than around 0.1 kg.

Each plane change amount resulted in similar trends for the change in propellant with respect to the thrust duration. The 15 deg plane change is represented in the following figures.

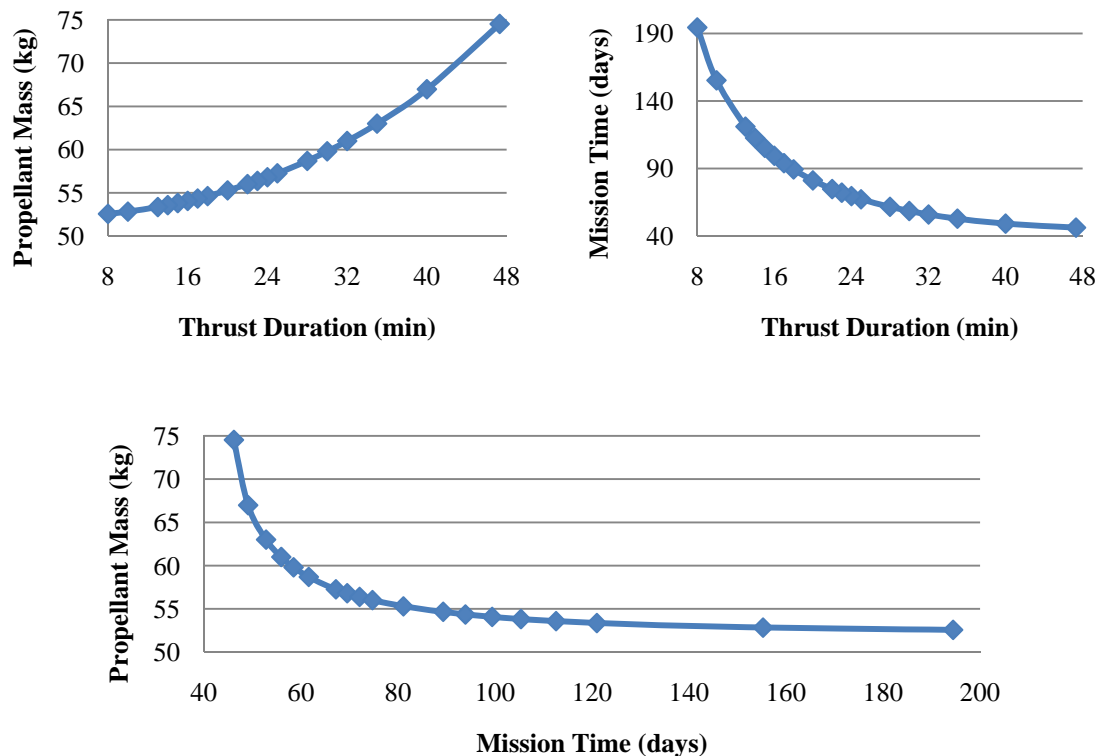


Figure 14. Two Burn 15 deg Plane Change

Figure 14 displays the values for a purely electric propulsion plane change maneuver. The least propellant resulted from the shortest thrust duration as seen in the uppermost plot. That same thrust duration would take 190 days to complete the 15 deg maneuver. The final data point on the upper two plots represents the continuous thrust case. That point corresponded to the highest required propellant at around 75 kg.

The continuous thrust case achieved the final orbit with the least amount of time for the electric propulsion profiles. All the values in Table 3 were for the transfer where the position and velocity were continuously updated based on the thrust duration. The values listed matched up with those shown in Figure 10 - Figure 14.

Table 3. 15 deg Plane Change Values

	Mission Req's	Continuous Thrusting	40 Min Thrust Duration	30 Min Thrust Duration	20 Min Thrust Duration	18 Min Thrust Duration	15 Min Thrust Duration
Propellant Mass (kg)	< 80	74.53	66.98	59.80	55.28	54.94	66.08
Trip Time (days)	< 90	46.16	49.11	58.47	81.11	89.32	87.25
Delta V (m/s)	N/A	3146.6	2739.4	2376.9	2159.3	2128.3	2073.3

The thrust duration that resulted in the least required propellant was an 18 min thrust duration centered at each crossover point. The propellant required decreased as the thrust duration decreased. The 15 min thrust duration required more propellant than the 18 min thrust duration because the electric system alone was unable to achieve the plane change within the time constraint. The utilization of the chemical propulsion system to achieve the plane change within the time constraint caused the required propellant to increase.

IV. Conclusion

Using simplifying assumptions resulted in an under prediction of the trip time and propellant mass required for the maneuver. The location of the thrust for the electric propulsion segment had a large effect on the amount of propellant required. The larger the trip time constraint the lower the propellant required to perform the maneuver. The fastest the MMP system would be able to perform the mission within the mass constraint was in 37 days. The electric propulsion system required half the propellant of the chemical propulsion system to perform the plane change. Plane change missions are very costly, but using optimal thrust profiles results in a drastic reduction in propellant required.

References

- ¹Kluever, C., "Spacecraft Optimization with Combined Chemical-Electric Propulsion," *Journal of Spacecraft and Rockets*, Vol. 32, No. 2, 1994, pp. 378-380.
- ²Edelbaum, T.N., "The use of high- and low-thrust propulsion in combination for space missions," *Journal of the Astronautical Sciences*, Vol. 9, No 2, 1962, pp. 49-69
- ³Bate, Roger, Mueller, Donald, & White, Jerry. (1971). *Fundamentals of Astrodynamics*. Dover Pubns.
- ⁴Petter, M., Heilman, S., "Multiple Mode Propulsion system for Spacecraft Orbit Transfer, Maneuvering, and Attitude Control," BAA-06-02-PKTB. (24 April 2006).
- ⁵Humble, Ronald, Henry, Gary, & Larson, Wiley. (1995). *Space propulsion analysis and design*. Learning Solutions.
- ⁶Larson, W., and Wertz, J., *Space Mission Analysis and Design*. Space Technology Library. (El Segundo, CA) 1999.
- ⁷Curtis, H., *Orbital Mechanics for Engineering Students*. Elsevier Ltd., 2005.